Problem Set # 1

1. **Buckingham Π-Theorem.** Geoffrey Taylor, a famous Cambridge fluid mechanic, annoyed the US government by doing the following analysis. The question he answered: What was the yield (in kilotons of TNT) of the first atomic blast (in the New Mexico desert in 1945)? Declassified pictures, which even had a scale bar, gave the following data on the radius of the explosion at various times:

t (ms)	R (m)
3.26	59.0
4.61	67.3
15.0	106.5
62.0	185.0

   (a) Identify the relevant parameters which determine the problem and use the Buckingham Π-Theorem to determine the number of non-dimensional groups.

   (b) Give an expression of the yield of the atomic blast.

   (c) Use the above given data to estimate the yield of the blast. (Is this possible without further knowledge of the underlying physics?)

   (d) Compare the result to the exact value of E $\approx$ 20 kilotons of TNT, knowing that one gram of TNT releases roughly 4kJ of energy.

   Further reading: The Formation of a Blast Wave by a Very Intense Explosion. II. The Atomic Explosion of 1945., Proceedings of the Royal Society of London. Series A, Mathematical and Physical 201(1065): 175186 (22 March 1950)

2. **Isotropic Decaying Turbulence.** In Direct Numerical Simulation (DNS), the Navier-Stokes equations are solved without any further modeling assumptions. Results from DNS are often used to study turbulence because of the complete description of the velocity field and the corresponding velocity statistics. Isotropic decaying turbulence is a particularly simple flow, in which all turbulence statistics are independent of space and even direction. Throughout this course, we will use a velocity field from a DNS of isotropic turbulence to explore the concepts discussed during lecture and gain experience analyzing turbulent flows.

   The simulation was done for a box of dimensions $2\pi \times 2\pi \times 2\pi$ using $256 \times 256 \times 256$ grid points. Periodic boundary conditions were used in all three directions. The data contains the three components of velocity and the pressure.

   (a) Using these data, plot a 1D profile of velocity versus coordinate along a line. You are free to choose any line you want (for e.g. $(i, j = 10, k = 10)$). By looking at the plot, visually estimate:

      i. the magnitude of the largest velocity fluctuations and the length over which these fluctuations occur.

ii. the magnitude of the smallest velocity fluctuations and the length over which these fluctuations occur.

(b) How would your length scales in part (a) change if the Reynolds number were increased?

(c) Compute the mean and variance of the three velocity components. If the data field were exactly isotropic, how would the variances of the velocity components be related?

3. **Turbulent Jet Photo and Video Contest.** Participation in the photo contest is voluntary. To participate, take a photo or video of a turbulent jet and estimate the Reynolds number. Make sure you get good resolution so that you see the turbulent structures on the photo. A PRIZE (hardcover copy of Milton van Dyke's "Album of Fluid Motion") will be given for the best original photograph and video. The photo can be composed of single photos at different conditions. Email the photo or video to the TA and Prof. Pitsch, or, if it is too large, bring a memory stick to class.

Example: Take a transparent container of as large as possible capacity. If you do not have a large transparent container, try to use a large non-transparent container. Fill it with water, and let it stand until the motion ceases (several minutes). Pour in about 1 ml of colored fluid (e.g. milk or juice; soy sauce doen't work very well) and observe the downward-going jet. The motion is mainly due to the momentum of the injected liquid, but depending on the liquids, buoyancy can play some part, as it does in clouds. Estimate the Reynolds number

$$\mathrm{Re} = \frac{uD}{\nu}\,,$$

where u is the jet velocity at impact (estimate from the balance of potential energy at the beginning and kinetic energy at impact), D is the jet diameter at impact (2 mm or whatever you think it is), and ν is estimated to be $\nu = 10^{-6}\,\mathrm{m}^2/\mathrm{s}$.

You can try to vary the Reynolds number by pouring from different heights. You can also have the water swirling before you pour the jet.

Notice how similar the motion is to that of smokestack or steam plumes or other very large-scale turbulence phenomena, which obviously occur at much higher Reynolds number. Because turbulence mixes matter and momentum much more quickly than viscosity and molecular diffusivity do, turbulent processes are nearly independent of viscosity and hence the Reynolds number.